PAPER

Joint Channel and Network Decoding for XOR-Based Relay in Multi-Access Channel

Suhua TANG[†*], **Jun CHENG**[††], *Members*, **Chen, SUN**[†††], *Nonmember*, **Ryu MIURA**[†],
and **Sadao OBANA**[†], *Members*

SUMMARY In this paper network coding based relay for multi-access channel is studied. In the system, two nodes send messages to a common access point (AP). A relay assists the two nodes by forwarding a network coded version of the messages. The AP performs joint channel and network decoding to recover the two original messages from three received signals. Two schemes, soft network coding (SoftNC) and turbo network coding (TurboNC), both focusing on bitwise exclusive or (XOR) based network coding, are proposed to salvage messages from erroneous signals. SoftNC is simple and backward compatible with existing protocol stack of wireless networks, and reduces packet errors by maximal ratio combining (MRC). TurboNC improves channel efficiency by letting the relay node transmit only parity check bits of the interleaved XORed message, where reliability is retained by iterative decoding. Simulation results show that compared with the network-layer path diversity scheme [8], both SoftNC and TurboNC greatly improve the reliability, and TurboNC also achieves a much higher throughput. The proposed schemes are suitable for improving the performance of wireless local area networks (WLAN).

key words: *relay, network coding, joint decoding, maximal ratio combining, iterative decoding*

1. Introduction

In wireless communications a signal transmitted by the sender usually arrives at the receiver via multiple paths due to reflection, diffraction and scattering. These signals have different phases and may enhance or cancel each other, which greatly degrades communication quality in times of deep fading.

Spatial diversity, exploiting the property of independent wireless propagation, is an effective method to combat fading. Such a consensus has inspired extensive research on relay theory, which exploits the broadcast nature of wireless medium. In the plain relay model, a relay node can amplify/decode and forward a message to reduce the outage probability. A relay scheme based on coded cooperation was introduced in [1].

The idea of network coding was originally proposed by Ahlswede et al. [2] to enhance the capacity of wired networks. This idea was extended later to wireless networks to enable efficient relay [3], either in the network-layer with decode-and-forward [4] or in the physical layer with amplify-and-forward [5]–[7]. The common characteristics of these schemes are to exploit a priori knowledge to reduce the number of transmissions and improve the total capacity of the bidirectional path.

Bitwise exclusive or (XOR) based network coding was widely used in many practical protocols [4] to increase network capacity due to its simplicity for implementation. Recently it was also exploited to improve reliability of wireless transmission. Chen et al. suggested realizing path diversity with network coding [8]. A similar scheme was proposed for reducing packet loss in the application layer [9]. In both schemes network coding is separated from channel coding.

Joint channel and network coding was also explored to further improve the performance of network coding. Hausl et al. suggested decoding network coded messages by constructing a parallel concatenated convolutional coding (PCCC) structure [10], [11]. A network coding scheme other than XOR was studied and therefore the decoding scheme cannot be directly applied to XOR-based network coding. In [12] the XOR operation at the relay node is done on the signals instead of the decoded messages to reduce the complexity at the cost of signal-to-noise ratio (SNR) degradation. However, how to perform joint decoding at the receiver was not discussed.

In XOR-based network coding, information of two flows is mixed together. By exploiting accurate a priori information, network coding works well in the bidirectional communication scenarios. In a multi-access channel, there is no perfect a priori knowledge at the receiver. How to separate the information and enable joint decoding is a big challenge.

In this paper we study XOR-based network coding for relay in multi-access channel. In the system two nodes send messages to a common access point (AP). A relay helps the two nodes by forwarding a network-coded version of original messages to the AP. The AP performs joint channel and network decoding to recover

---

Manuscript received January 1, 2009.
Manuscript revised January 1, 2009.
Final manuscript received January 1, 2009.

[†] ATR Adaptive Communications Research Laboratories, 2-2-2 Hikaridai “Keihanna Science City” Kyoto 619-0288 Japan.

[††] Department of Intelligent Information Engineering and Sciences, Doshisha University, 1-3 Tatara Miyakodani, Kyotanabe, Kyoto 610-0321 Japan.

[†††] Ubiquitous Mobile Communications Group, National Institute of Information and Communications Technology (NICT), YRP-1 Bldg., Hikarinooka 3-4, Yokosuka 239-0847, Japan.

[*] E-mail: shtang@atr.jp

the two original messages from three signals received from relay and two nodes. Under this common model, two schemes, soft network coding (SoftNC) and turbo network coding (TurboNC), are proposed. For the *backward compatibility*, a relay node in SoftNC works as usual [4] and forwards both information bits and parity check bits of the XORed message. To further improve *channel efficiency*, a relay node in TurboNC interleaves the XORed message and forwards only its parity check bits. At the receiver side, maximal ratio combining (MRC) is applied in SoftNC compared with the separate decoding in [8]. TurboNC adopts iterative decoding. It generates fewer parity check bits at the relay node and has higher throughput in comparison with the method in [11]. Simulation results show that compared with the scheme in [8], both SoftNC and TurboNC greatly improve reliability and TurboNC also achieves a much higher throughput. Note that although SoftNC is inferior to TurboNC, we still propose it for the reason of the backward compatibility. In practical networks, the APs, as infrastructures, can be updated to support both SoftNC and TurboNC. But various versions of relays, as mobile terminals, will co-exist. The old relays suggested in [4], [8], which don't support TurboNC, will benefit from the backward compatibility of SoftNC. The two schemes are quite suitable for improving performance of wireless local area networks (WLAN) and they can be extended to distributed multi-hop networks such as inter-vehicle communications.

The rest of this paper is organized as follows. We present the network coding based relay model in Sect. 2 and the preliminary of log-likelihood algebra in Sect. 3. On this basis we propose the joint coding/decoding schemes in Sect. 4 and evaluate their performance by simulation in Sect. 5. Finally we conclude this paper with Sect. 6.

2. System Model

Figure 1 shows the system model. Nodes A and B respectively send messages X_{AU} and X_{BU} to the common access point D and relay R in two time slots. Each message carries its cyclic redundant check (CRC) and has the same length. When the two messages arriving at R both pass CRC check after channel decoding, R encodes them and transmits† the coded message in a third time slot†† together with an out-of-band signal informing D of this network coded transmission. Each node, including the relay, uses a fixed power for transmission.

In the proposed model systematic convolutional codes with channel coding operation $\Gamma(\cdot)$ are used. For the simplicity of description, binary phase-shift keying (BPSK) modulation is assumed. The interleaving operation $\Pi(\cdot)$ and the corresponding de-interleaving $\Pi^{-1}(\cdot)$ are used in TurboNC. The combination of interleaving and channel coding is $\Gamma'(\cdot) = \Gamma(\Pi(\cdot))$.

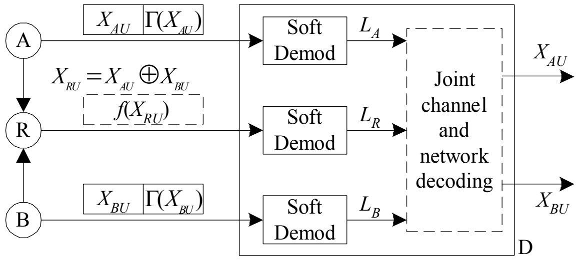


Fig. 1 Joint channel and network coding in multi-access channel.

An original message X_{iU} from A or B is channel coded according to Eq.(1) and its parity check part is X_{iC}.

$$X_i = [X_{iU}, X_{iC}], \quad X_{iC} = \Gamma(X_{iU}), \quad i = A, B. \tag{1}$$

When R correctly decodes both X_{AU} and X_{BU}, it combines the two messages by XOR and get $X_{RU} = X_{AU} \oplus X_{BU}$. Then it transmits the jointly coded version $X_R = f(X_{RU})$, where $f(\cdot)$ is an encoding function.

The coded packets X_A, X_B and X_R with bit streams $x_A(t)$, $x_B(t)$ and $x_R(t)$ are transmitted to D from A, B, R, respectively. The signals of BPSK modulation are

$$\varphi_i(t) = 2 * x_i(t) - 1, \quad x_i(t) \in \{0, 1\}, \quad i = A, B, R. \tag{2}$$

Assume that links AD, BD and RD in Fig. 1 experience block fading. They have channel gains h_A, h_B, h_R, and additive white Gaussian noise (AWGN) $n_A(t)$, $n_B(t)$, $n_R(t)$ with zero-mean and equal variance σ^2. D receives signals from A, B and R, as shown below.

$$s_i(t) = h_i \varphi_i(t) + n_i(t), \quad i = A, B, R. \tag{3}$$

Then D performs joint channel and network decoding with the three received signals and their channel gains measured at the timing of reception. Messages passing CRC check are regarded as being successfully received.

3. Preliminaries of Log Likelihood Algebra

In the proposed joint decoding schemes in Sect. 4 we will use log likelihood algebra [13]. We briefly describe it in this section.

Usually coded bits $x_i(t)$ have equal probability of being 0 or 1. Then, bit log-likelihood ratio (LLR) of received signals can be calculated as follows [13]:

$$L_i(t) = \ln \frac{P(x_i(t) = 1 | s_i(t))}{P(x_i(t) = 0 | s_i(t))} = \frac{2 h_i s_i(t)}{\sigma^2}. \tag{4}$$

LLR can serve as the soft output of a demodulator.

---

† Alternatively, R may transmit the coded message at the request of D.

†† Only half of the third time slot is used in TurboNC.

When $X_R = X_A \oplus X_B$, LLR of X_R can be estimated from $L_A(t)$ and $L_B(t)$ by the following log-likelihood algebra [13].

$$L'_R(t) = L_A(t) \boxplus L_B(t) = \ln \frac{\exp(L_A(t)) + \exp(L_B(t))}{1 + \exp(L_A(t) + L_B(t))} \approx (-1) \cdot \text{sign}[L_A(t)] \cdot \text{sign}[L_B(t)] \cdot \min(|L_A(t)|, |L_B(t)|). \quad (5)$$

Meanwhile, the XOR operation has some special property shown in the following equation.

$$X_R = X_A \oplus X_B \Longrightarrow X_A = X_B \oplus X_R, \quad X_B = X_A \oplus X_R. \quad (6)$$

Equation(6) indicates that any message is an XORed version of the other two. Then according to Eq.(5) LLR of any message can be estimated from the other two.

Consider the extreme case where D somehow correctly receives one coded message, e.g. X_R. When $x_R(t) = 1$, $L_R(t) = \infty$ and $L_R(t) \boxplus L_A(t) = -L_A(t)$. When $x_R(t) = 0$, $L_R(t) = -\infty$ and $L_R(t) \boxplus L_A(t) = L_A(t)$. Then from $L_B(t)$ and $L_A(t)$, the LLR of X_A and X_B can be estimated as $L'_A(t)$ and $L'_B(t)$ respectively, as follows:

$$\begin{aligned} L'_B(t) &= (-1)^{x_R(t)} L_A(t), \text{ if } x_R(t) \text{ is known } \textit{a priori}, \\ L'_A(t) &= (-1)^{x_R(t)} L_B(t), \text{ if } x_R(t) \text{ is known } \textit{a priori}. \end{aligned} \quad (7)$$

Estimating LLR of the XORed message with Eq.(5) results in SNR loss. In Eq.(7) the estimated LLR has the same SNR as the original LLR by exploiting the accurate a priori information. This observation inspires us to design new schemes to further exploit diversity.

4. Proposed Relay Schemes

In Fig. 1 the encoding scheme at R is determined by a function $f(\cdot)$. In the following we study SoftNC and TurboNC, two schemes corresponding to two encoding functions $f(\cdot)$, and their decoding methods.

4.1 Soft Network Coding (SoftNC)

$f(X_{RU}) = [X_{RU}, \Gamma(X_{RU})]$ is used in SoftNC in Fig. 1. Both the information bits and the parity check part of the XORed message are transmitted from R to D. It was mentioned in [8] that the two original messages X_{AU} and X_{BU} can be correctly decoded at D if any two of the three messages X_{AU}, X_{BU} and X_{RU} are correctly received. In the following we discuss how to decode the original messages when only one of X_{AU}, X_{BU} and X_{RU} is correctly received. To keep SoftNC simple, no further operation is done when all messages are erroneous. According to Eq.(6), any message is an XORed sum of the other two messages. Assume, without loss of generality, that D correctly decoded X_{RU}. Then D decodes the other two messages by applying MRC [†].

Since $X_{RU} = X_{AU} \oplus X_{BU}$ contains network coded information bits, by applying the linear property of channel coding and XOR, the order of XOR and channel coding can be exchanged [14].

$$\Gamma(X_{AU} \oplus X_{BU}) = \Gamma(X_{AU}) \oplus \Gamma(X_{BU}). \quad (8)$$

Then $X_R = [X_{RU}, X_{RC}]$, $X_{RC} = \Gamma(X_{RU})$, is an XORed sum of X_A and X_B, i.e., $X_R = X_A \oplus X_B$.

With the decoded X_{RU}, D locally generates $X_{RC} = \Gamma(X_{RU})$ and regards $X_R = [X_{RU}, X_{RC}]$ as a priori knowledge. According to the LLR algebra in Eq.(7), D calculates $L'_A(t)$ as an estimation of $L_A(t)$ from $L_B(t)$, and applies combining diversity as follows:

$$L''_A(t) = L'_A(t) + L_A(t) \quad (9)$$

The above equation is exactly MRC, as is explained below. Because $x_B = x_A \oplus x_R$, under BPSK modulation $\varphi_B(t) = (-1)^{x_R(t)} \varphi_A(t)$. With this relation and Eq.(4), Eq.(9) can be simplified as follows:

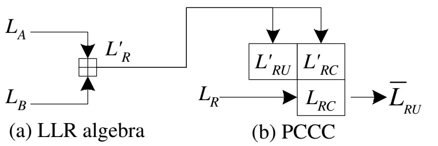
$$\begin{aligned} L''_A(t) &= \frac{2h_B}{\sigma^2} s'_A(t) + \frac{2h_A}{\sigma^2} s_A(t), \\ s'_A(t) &= h_B \varphi_A(t) + (-1)^{x_R(t)} n_B(t). \end{aligned} \quad (10)$$

Except the noise part it looks as if D receives another copy of $\varphi_A(t)$ from B. Two copies of the same message (X_A) are combined together according to their channel gains. In addition, the LLR after combination corresponds to a signal with an SNR equaling the sum of SNR of $s_A(t)$ and $s_B(t)$. In a similar way $L''_B(t)$ can be generated for X_B. It is worth pointing out that $L''_A(t)$ and $L''_B(t)$ are not independent. They contain the same information, corresponding to signals with the same SNR. From either of the combined LLR, e.g. $L''_A(t)$, message X_{AU} can be channel decoded and then X_{BU} can be network decoded according to Eq.(6).

The combination in Eq.(9) can be done according to either LLR from soft demodulation in Eq.(4), or LLR from a soft-input soft-output (SISO) channel decoder [17]. In the former case, the combined LLR is directed to a channel decoder to generate decoding output.

Application of MRC in SoftNC is limited to the case where one message is correctly decoded at the receiver. Packet errors occur when none of the three messages is correctly decoded. Although it is still possible to combine signals in such cases, this is not done after taking into account the tradeoff between the decoding cost and benefit, as explained below: When all messages are erroneous Eq.(7) cannot be used. Instead, with L_R as the a priori signal, the receiver gets a noisy estimation of L_A from L_B by Eq.(5), and combines it with L_A according to Eq.(9). The benefit of this combining is really marginal because the estimated LLR is noisy and neither interleaving nor iterative decoding is used in SoftNC for the purpose of simplicity. Therefore in such cases signal combining is not used and packet errors occur.

[†] As an alternative in a real system, the receiver may choose to first decode the signal with highest SNR and then combine the other two signals.



(a) LLR algebra (b) PCCC

Fig. 2 Turbo decoding of the XORed message, DEC3 in Fig. 3.

In SoftNC, network coding at the relay node is done in the network layer. Only network software is changed and network coding works on existing wireless networking protocols and hardware. An AP with the capability of SoftNC performs joint decoding, or otherwise performs separate decoding as in [8]. This backward compatibility enables incremental deployment.

4.2 Turbo Network Coding (TurboNC)

In this scheme, $f(X_{RU})$ in Fig. 1 equals $\Gamma'(X_{RU})$. In other words, the network coded message X_{RU} is first interleaved and then only its parity check part is actually transmitted. We call this scheme turbo network coding (TurboNC). It is obvious that the relay efficiency is improved by avoiding the transmission of information bits at R. The following iterative decoding also retains low packet error rate (PER) thanks to the interleaving. Each iteration involves the decoding of the XORed message and decoding of the original messages.

4.2.1 Decoding the XORed Message

Receiver D cannot directly decode X_{RU} as usual since its information bits are not transmitted. Fortunately LLR of X_R can be estimated from LLRs of X_A and X_B according to Eq.(5). In Fig. 2, LLR of X_A and X_B is respectively calculated according to Eq.(4). Then by Eq.(5), $L'_R = [L'_{RU}, L'_{RC}]$, is calculated from L_A and L_B and used as an estimation of the network coded message $X_R = [X_{RU}, \Gamma(X_{RU})]$, as shown in Fig. 2(a). Meanwhile, D also calculates $L_R = L_{RC}$, the LLR of the parity check part $\Gamma'(X_{RU})$ received from R. Since the systematic convolutional code is used, in Fig. 2(b) the PCCC structure of X_{RU} is formed. Then X_{RU} is decoded by several iterations of the turbo decoding [15], which further uses the SISO channel decoding algorithms such as BCJR [16] or LogMAP [17].

The turbo decoder outputs the decoded message X_{RU} if the decoding is successful. Otherwise it outputs the LLR value $\overline{L}_{RU}$ of X_{RU}. Either of the output will be used in the decoding of the original messages.

4.2.2 Decoding the Original Messages

SISO is also used in decoding the original messages. In Fig. 3 $L_A(t)$ and $L_B(t)$, LLR of the directly received signals, are inputted to the SISO channel decoders DEC1 and DEC2. DEC3 corresponds to the procedure in Fig. 2. DEC1, DEC2 and DEC3 also have extra inputs, $L'_{AU}(t)$, $L'_{BU}(t)$ and $L'_{RU}(t)$, a priori knowledge of the information bits which is initiated to zero. The soft decoding output of DEC1, DEC2, DEC3 are $\overline{L}_{AU}(t)$, $\overline{L}_{BU}(t)$ and $\overline{L}_{RU}(t)$, respectively. According to Eq.(5), the extrinsic information,

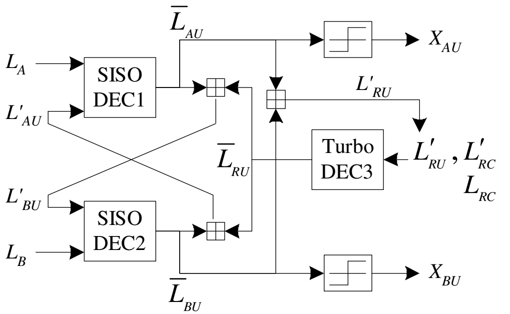


Fig. 3 Iterative decoding in TurboNC scheme.

$$\begin{aligned} L'_{AU}(t) &= \overline{L}_{BU}(t) \boxplus \overline{L}_{RU}(t), \\ L'_{BU}(t) &= \overline{L}_{AU}(t) \boxplus \overline{L}_{RU}(t), \\ \text{and } L'_{RU}(t) &= \overline{L}_{AU}(t) \boxplus \overline{L}_{BU}(t), \end{aligned} \tag{11}$$

are extracted and used as a priori information in the next iteration in DEC1, DEC2 and DEC3.

4.2.3 Decoding Steps of TurboNC

Receiver D first performs separate channel decoding to recover the two original messages X_{AU} and X_{BU}. If this fails, D applies the TurboNC scheme to jointly decode messages as follows: D estimates L'_R from L_A and L_B, and decodes X_{RU} with L'_R and L_{RC} by DEC3. L'_{AU} and L'_{BU} are calculated according to Eq.(11). Then DEC1 and DEC2 in Fig. 3 generate new $\overline{L}_{AU}$ and $\overline{L}_{BU}$. From the two LLRs, a new estimation of X_{RU}, L'_{RU}, is calculated. It is used together with the initial L'_{RC} and L_{RC} in DEC3 in next iteration. Note that if X_{RU} is correctly decoded, the decoding of X_{AU} and X_{BU} can be reduced to MRC stated in Sect. 4.1. In this way the computation cost can be decreased.

4.2.4 Some Discussions

When neither X_{AU} nor X_{BU} is correctly decoded from signals received over direct links, the LLR algebra in Figs. (2-3) follows Eq.(5), which generates an estimated LLR with lower SNR. The decrease of SNR in the latter, however, can be supplemented by the turbo decoding. When X_{AU}, X_{BU} and X_{RU} are respectively channel coded with a coding rate of 1/2, the PCCC for X_{RU} shown in Fig. 2 has a coding rate of 1/3, which is powerful enough to decode X_{RU} from signals with low SNR.

When either of the original messages, X_{AU} or X_{BU}, is directly decoded, the rest decoding can be performed in a different form, as shown in Fig. 4. Assume,

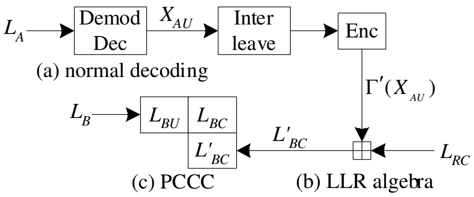


Fig. 4 Turbo decoding in a different form when one of the original messages can be decoded.

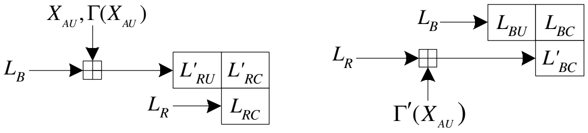


Fig. 5 Comparison of two PCCC structures.

without loss of generality, that the message which D correctly decodes is X_{AU}. Then D interleaves this message and performs channel coding again to get $\Gamma'(X_{AU})$. XOR, interleaving and channel coding all are linear. Therefore,

$$\Gamma'(X_{RU}) = \Gamma'(X_{AU}) \oplus \Gamma'(X_{BU}) \tag{12}$$

is also network coded. With the locally generated $\Gamma'(X_{AU})$ as a priori information and according to Eq.(7), L'_{BC} can be calculated as an estimation of $\Gamma'(X_{BU})$ from L_{RC}, the LLR of $\Gamma'(X_{RU})$. Then in Fig. 4(c), a PCCC structure of X_{BU} can be formed and this message can be decoded with the standard turbo decoding. This special case is reduced to distributed turbo coding [18].

According to the above description, when either of the original messages, e.g. X_{AU}, is directly decoded, the receiver D has two choices, either to recover the network coded message X_{RU} by turbo decoding in Fig. 2, or to directly recover the other original message X_{BU} by turbo decoding in Fig. 4. As shown in Fig. 5, in either case L_A, the LLR of the correctly decoded message, is not used. In both PCCC structures, the upper row comes from L_B and the lower row comes from L_R. Although the LLR algebra is involved in different stages, with X_{AU}, $\Gamma(X_{AU})$ and $\Gamma'(X_{AU})$ as accurate a priori knowledge the operation in Eq.(7) generates an estimated LLR with the same SNR. The LLRs in both PCCC contain the same information. Therefore the two procedures actually have the same decoding probability, as verified by simulation.

4.3 Comparison among Different Schemes

Table 1 shows a comparison among schemes discussed before. TurboNC+ is a variant of TurboNC where systematic bits are transmitted at R. NetCod is a scheme introduced in [8]. Although channel coding is not addressed in [8], for the purpose of fair comparison channel coding is involved in NetCod. Transmissions over direct links are the same in all schemes, $[X_{AU}, \Gamma(X_{AU})]$ by A and $[X_{BU}, \Gamma(X_{BU})]$ by B. In schemes other than Direct, R also transmits a coded message, as shown in the second column of Table 1.

Table 1 A comparison among different schemes. Coding rate is measured in the normal mode.

Scheme	Transmission by relay	Total coding rate
Direct	none	r
NetCod	$X_{RU}, \Gamma(X_{RU})$	$2/3 \cdot r$
SoftNC	$X_{RU}, \Gamma(X_{RU})$	$2/3 \cdot r$
TurboNC	$\Gamma[\Pi(X_{RU})]$	$2/(3-r) \cdot r$
TurboNC+	$\Pi(X_{RU}), \Gamma[\Pi(X_{RU})]$	$2/3 \cdot r$

Let r be the coding rate of the channel coding operation $\Gamma(\cdot)$. Assume that all coded messages, generated at both sources (A and B) and the relay node (R), form a super code. This code is used for transmitting the two original messages. The total coding rate of this super code can be easily calculated in the normal mode where R transmits network coded message regardless of the transmission status over direct links. The total coding rate under different schemes is listed in the third column of Table 1. It is obvious that among the relay schemes, TurboNC has the largest coding rate.

5. Numerical Evaluation

In this section we evaluate the proposed schemes using Monte-Carlo simulations. Each plain message including its CRC consists of 2400bits. Messages are coded by a 4-state recursive systematic convolutional (RSC) code with the code rate 1/2 and the generator matrix (1, 5/7). A random permutation matrix is used in TurboNC. The received signals are decoded by schemes listed in Table 1, respectively. Results of TurboNC+ are also given as a reference. In the decoding stage, constant Log-MAP is used. The number of iterations in TurboNC is set to 18. We say that packet errors occur if two original messages A and B cannot be both correctly decoded at D. In other words, X_{AU} or X_{BU}, or both, are erroneous after decoding.

The simulation focuses on the multi-access scenario in Fig. 1. Links experience block Rayleigh fading. A and B are close to each other. Positions of A/B and D are fixed unless otherwise specified. Distance between A/B and D equals d_{AD}. R lies between A/B and D and has an adjustable distance d_{R-AB} to A/B.

5.1 Effect of Relay Position

We first demonstrate how the position of the relay node affects the system performance. Average SNR of links AD/BD is fixed at 5dB. Adjusting the position of R between A/B and D changes the normalized distance d_{R-AB}/d_{AD}. Average SNR of links AR, BR and RD is calculated from the normalized distance d_{R-AB}/d_{AD} according to two-ray model [20] with the path loss exponent (equalling 4 in the simulation).

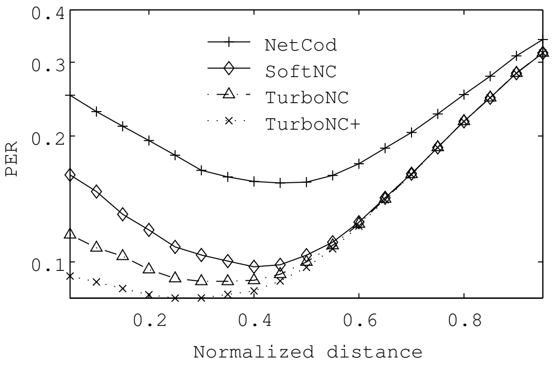


Fig. 6 PER under different normalized distances (d_{R-AB}/d_{AD}) between relay and mobile nodes (average SNR of direct links is fixed at 5dB).

Figure 6 shows the PER under different distances. When R is far from A/B, the SNR of the relay links AR/BR is low. In all schemes, when R cannot correctly decode either X_{AU} or X_{BU}, it does not forward the network coded message to D. In such extreme cases all relay schemes have the same poor performance, which depends on the quality of direct links AD/BD. A decrease in the distance between R and A/B improves the delivery rate over links AR/BR. Then with a higher probability R forwards the network coded message and contributes to the decoding at D. Therefore PER of these schemes reaches the minimum when R lies to the left of the middle point (normalized distance equals 0.5). When the messages are successfully transferred over links AR/BR, in the proposed schemes, the following joint decoding reduces message errors over the multi-access channel from $A/B/R$ to D. As a result the minimal PER of the two proposed schemes is much less than that of NetCod. When R is not very close to A/B (not too far from D), D can decode the XORed message (either direct plain channel decoding in SoftNC or turbo decoding in TurboNC) with a high probability and then decoding of the original messages is done by MRC. Therefore, TurboNC is only a little better than SoftNC. As R gets nearer to A/B and farther away from D, the probability with which D directly decodes the XORed message in SoftNC decreases, but the iterative decoding of TurboNC in Fig. 3 helps reduce the total PER.

5.2 PER under Two Typical Scenarios

In the following we consider two typical scenarios. In the first scenario A and B are near to each other. R lies exactly in the middle of A/B and D and only serves as a relay node. Links between R and other nodes A, B, D have an average SNR 12dB[†] higher than direct links AD/BD. In the second scenario A, B and R are near to each other and AR/BR has an average SNR of 40dB[††]. In this scenario all nodes can mutually cooperate and each node can help the other two.

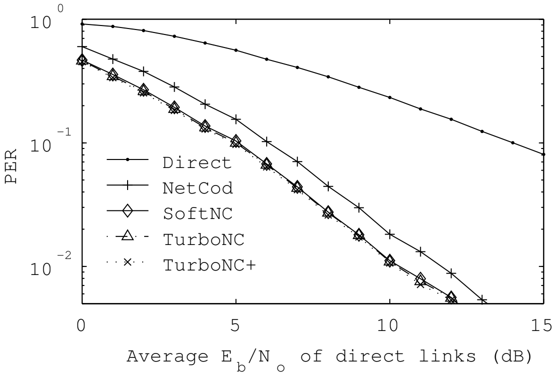


Fig. 7 PER performance in the first scenario (relay lies in the middle of mobile nodes and AP).

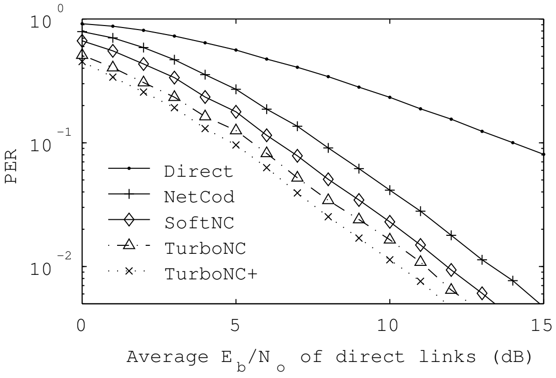


Fig. 8 PER performance in the second scenario (mutual cooperation among mobile nodes).

Figures 7-8 show PER with respect to E_b/N_o (SNR per bit) under the two scenarios respectively. In either scenario PER decreases as SNR of direct links AD/BD increases. In the first scenario D decodes the XORed message with a relatively high probability. Then D decodes the rest messages by applying MRC according to Eq.(9). The main factor that arbitrates the total PER is the signals over direct links. Therefor, PER curves of SoftNC, TurboNC and TurboNC+ nearly overlap, coinciding with Fig. 6 where the normalized distance equals 0.5. SoftNC and TurboNC each provide about 1 dB gain to NetCod. In the second scenario three links AD, BD and RD have the same average SNR. The probability with which D directly decodes the XORed message in SoftNC is relatively low. In TurboNC, the XORed message is decoded with a relatively high probability by turbo decoding. Even this fails the following iterative decoding can further salvage some messages.

[†] This SNR is calculated according to path loss exponent (equalling 4) and the normalized distance (equalling 0.5).

[††] The SNR is selected to ensure that neighboring nodes can hear from each other packets transmitted at the highest rate (54Mbps in IEEE 802.11).

Table 2 TurboNC transmission and its throughput under ARQ mode (code rate $r = 1/2$).

D sends request	No	Yes		Yes		
R transmits NC msg	No			Yes		
#decoded msg at D	2	1	0	2	1	0
Achieved throughput	1	1/2	0	2/2.5	1/2.5	0

Therefore superiority of TurboNC becomes very obvious. Its gain to NetCod increases to 2.2 dB. TurboNC+ has higher gain than TurboNC because R transmits more bits in TurboNC+.

5.3 Throughput under ARQ Mode

PER is reduced in relay schemes at the cost of extra transmissions. A fair comparison among all schemes in terms of channel efficiency must be based on the actual throughput, which takes both PER and overhead into account. In this paper *throughput* is defined as the number of messages transmitted from sources to destination in a time slot. According to the third column of Table 1, Direct has the highest coding rate. Accordingly it has higher throughput than relay schemes in the normal node when SNR of direct links is high enough.

In the following we compare the throughput of different schemes under automatic repeat-request (ARQ) mode, where R transmits only when it receives a request from D indicating that errors occurred over direct links. TurboNC transmission in the ARQ mode and the calculation of its throughput† (code rate $r = 1/2$) are given in Table 2. There are two main cases: (i) R does not transmit, either because there is no request from D or R itself fails to decode the original messages, and (ii) R transmits the network coded message at the request of D. 2 time slots are taken in case (i) and 2.5 time slots are taken in case (ii). According to the number of messages D (directly or jointly) decodes successfully, the achieved throughput is 1, 1/2 and 0 in case (i) and 2/2.5, 1/2.5 and 0 in case (ii). Then throughput under all cases is averaged according to their probability. Average throughput of other relay schemes is calculated in a similar way.

Fig. 9 compares average throughput achieved by different schemes in the second scenario described in Sect. 5.2. All relay schemes have higher throughput than the Direct scheme and TurboNC always outperforms other relay schemes. This also justifies the decision that systematic bits not be transmitted at the relay node in TurboNC, as compared with TurboNC+.

Working well in the ARQ mode is also an advantage of TurboNC over the scheme in [11]. When errors occur over direct links, the receiver correctly receives one original message with a certain probability. Assume, without loss of generality, that over direct links D recovers X_{AU} but X_{BU} is erroneous. In such cases, transmitting coded bits of X_{RU} at R is equivalent to transmitting coded bits of X_{BU}, as discussed in Sect. 4.2.4. Then all information transmitted by R contributes to the decoding of X_{BU}, while this is not the case in [11].

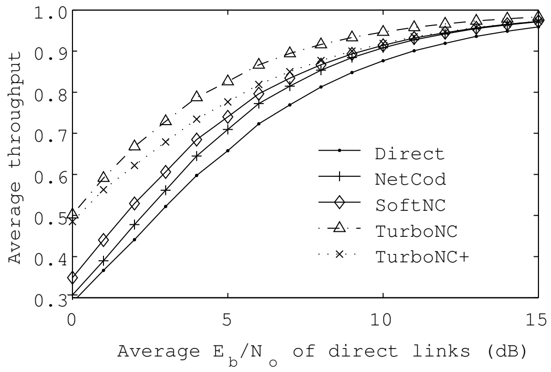


Fig. 9 Average throughput (number of messages per slot) in ARQ mode in the second scenario (mutual cooperation).

6. Conclusion and Future Work

Network coding-based relay model was studied in the multi-access channel and two joint channel and network coding/decoding schemes (SoftNC and TurboNC) were proposed for XOR-based network coding. The two schemes aim to salvage messages from erroneous signals and have following characteristics: (i) Decoding the network coded message directly (in SoftNC) or by turbo decoding (in TurboNC), and, (ii) Using MRC (in SoftNC) or iterative decoding (in TurboNC) to decode the original messages. The proposed schemes have different capabilities in backward compatibility, complexity, reliability and channel efficiency. SoftNC is simple and backward compatible with existing protocol stack of wireless networks and enables incremental deployment. TurboNC is relatively complex, but is more efficient and more reliable. Both schemes are applicable to WLAN for improving channel efficiency. They also can be extended to enhance inter-vehicle communications, where reliable transfer is of great importance. In the future we will continue to study how to allocate power among nodes and relay and how to handle link asymmetry.

Acknowledgment

This work was supported by the National Institute of Information and Communications Technology (NICT), Japan.

References

[1] T. E. Hunter and A. Nosratinia, "Diversity through coded cooperation," IEEE Trans. on Wireless Communications, vol.5, no.2, pp. 283–289, 2006.

---

†ARQ signaling overhead is not taken into account.

[2] R. Ahlswede, N. Cai, S. Y. R. Li, and R. W. Yeung, "Network information, flow," IEEE Trans. on Information Theory, vol. 46, no.4, pp. 1204–1216, 2000.

[3] P. Larsson, N. Johansson, and K. E. Sunell, "Coded bi-directional relaying," IEEE VTC2006-Spring, vol.2, pp.851–855, 2006.

[4] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in the air: practical wireless network coding," ACM SIGCOMM'06, pp.243–254, 2006.

[5] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multi-hop network," IEEE VTC2006-Spring, vol.2, pp.588–593, 2006.

[6] S. L. Zhang, S. C. Liew, and P. P. Lam, "Physical-layer network coding," ACM MobiCom'06, 2006.

[7] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: analog network coding," ACM SIGCOMM'07, pp. 397–408, 2007.

[8] Y. D. Chen, S. Kishore, and J. Li, "Wireless diversity through network coding," IEEE WCNC'06, vol.3, pp.1681–1686, 2006.

[9] T. K. Chua and D. C. Pheanis, "QoS evaluation of sender-based loss-recovery techniques for VoIP," IEEE Network, vol.20, no.6, pp.14–22, 2006.

[10] C. Hausl and J. Hagenauer, "Iterative network and channel decoding for the two-way relay channel," IEEE ICC'06, vol.4, pp.1568–1573, 2006.

[11] C. Hausl and P. Dupraz, "Joint network-channel coding for the multiple-access relay channel," IEEE SECON'06, vol.3, pp.817–822, 2006.

[12] S. L. Zhang, Y. Zhu, S. C. Liew, and K. B. Letaief, "Joint design of network coding and channel decoding for wireless networks," IEEE WCNC'07, pp.779-784, 2007.

[13] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," IEEE Trans. Info. Theory, vol.42, no.2, pp.429–445,1996.

[14] L. Xiao, T. E. Fuja, J. Kliewer, and D. J. Costello, "Nested codes with multiple interpretations," 40th annual conference on Information Sciences and Systems, pp.851–856, 2006.

[15] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near shannon limit error-correcting coding and decoding: turbo codes," IEEE ICC'93, pp.1064–1070, 1993.

[16] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," IEEE Trans Inform. Theory, vol.20, no.2, pp. 284–287, 1974.

[17] P. Robertson, P. Hoeher, and E. Villebrun, "Optimal and suboptimal maximum a posteriori algorithms suitable for turbo decoding," Europ. Trans. Telecommun., vol.8, no.2, pp.119–125, 1997.

[18] B. Zhao and M. C. Valenti, "Distributed turbo coded diversity for the relay channel," IEE Electronics Letters, vol.39, no.10, pp.786–787, 2003.

[19] Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specification, IEEE Std. 802.11, 1997.

[20] A. Goldsmith, "Wireless Communications," Cambridge University Press, 2005.

Suhua Tang received the B.S. degree in Electronic Engineering in 1998 and the Ph.D. degree in Information and Communication Engineering in 2003 from University of Science and Technology of China. Since 2003 he is with ATR Adaptive Communications Research Laboratories. His research interests include signal processing, cross-layer design, mobile ad hoc networks and inter-vehicle communications. He is a member of IEEE.

Jun Cheng received the B.S. and M.S. degrees in telecommunications engineering from Xidian University, Xi'an, China, in 1984 and 1987, respectively, and the Ph.D. degree in electrical engineering from Doshisha University, Kyoto, Japan, in 2000. From 1987 to 1994, he was an Assistant Professor and Lecturer in the Department of Information Engineering, Xidian University. From 1995 to 1996, he was an Associate Professor in the National Key Laboratory on Integrated Service Network, Xidian University. In April 2000, he joined ATR Adaptive Communications Research Laboratories, Kyoto, Japan, where he was a Visiting Researcher. From August 2002 to June 2003, he was working as a Staff Engineer at the R&D Center, Panasonic Mobile Communications Co., Ltd. (formerly Wireless Solution Laboratories, Matsushita Communication Industrial Co., Ltd.),Yokosuka, Japan. From July 2003 to March 2004, he was a Staff Engineer at Next-Generation Mobile Communications Development Center, Matsushita Electric Industrial Co., Ltd., Yokosuka, Japan. In April 2004, he joined Doshisha University, Kyoto, Japan, where he is currently an Associate Professor in the Department of Intelligent Information Engineering and Sciences. During 2005-2008, he was a Visiting Researcher in ATR Wave Engineering Laboratories, Kyoto, Japan. His research interests are in the areas of communications theory, information theory, coding theory, array signal processing, and radio communication systems.

Chen Sun received the PhD degree in electrical engineering from Nanyang Technological University, Singapore in 2005. From November 2002 to March 2003, he was with ATR Adaptive Communications Research Laboratories, Japan as a student intern. From August 2004 to May 2008 he was a researcher at ATR Wave Engineering Laboratories, Japan. In June 2008, he joined Ubiquitous Mobile Communications Group, National Institute of Information and Communications Technology (NICT) Japan as an expert researcher working on cognitive radio and IEEE SCC41 1900.6 standardization. His research interests include cognitive radio systems, smart antennas, and cooperative communications. He is a member of IEEE.

Ryu Miura received the B.E., M.E., and Dr. Eng. degrees in Electrical Engineering from Yokohama National University, Yokohama, Japan, in 1982, 1984, and 2000, respectively. He joined Communications Research Lab (CRL), Ministry of Posts and Telecommunications, Tokyo, Japan in 1984. Since then he was involved in the research project on satellite communication systems. During 1991-1992, he was a visiting researcher in AUSSAT, Pty. Ltd. (now Optus, Pty. Ltd.), Sydney, Australia, where he was involved in R&D on mobile satellite communications using Japanese Engineering Test Satellite ETS-V. During 1993-1996, he was a senior researcher in ATR Optical and Radio Communications Research Labs, Kyoto, where he was involved in R&D on digital beamforming (DBF) antenna systems for mobile satellite communications. During 2001-2004, he managed the R&D on telecom payload in the Japanese High Altitude Platform Program "Skynet". During 2006-2007, he doubled as the Director of Singapore Office, where he managed the research project on maritime ITS (intelligent transport system) under the collaboration with the Institute for Infocom Research (IIR), Singapore. Now he is the Department Head of Smart Networks in ATR. He is a member of IEEE.

Sadao Obana received the B.E., M.E. and Dr. Eng. Degrees from Keio Univ. Tokyo, Japan, in 1976, 1978 and 1993 respectively. After joining KDDI (former KDD) in 1978, he was engaged in R&D in the field of packet exchange systems, network architecture, OSI (Open Systems Interconnection) protocols, database, distributed processing, network management and ITS (Intelligent Transport Systems). In 2004, he joined Advanced Telecommunication Research Institute International (ATR) and is now a director of Adaptive Communications Research Laboratories in ATR. His current research areas are wireless mobile ah-hoc network, ubiquitous wireless sensor network and ITS. He received an Award of Minister of Education, Culture, Sports, Science and Technology in 2001. Dr. Obana is a Fellow of Information Processing Society of Japan (IPSJ).